**Formal Geometry** **Name**_____
**Ch 5 Review Worksheet**

1. If a triangle has two sides with lengths of 6 cm and 19 cm. Which length(s) below could **not** represent the length of the third side? Choose all that apply.

**A.** 7 cm **C.** 25 cm
**B.** 13 cm **D.** 22 cm

The length of each side of a triangle must be

- Greater than the difference of the other two, and
- Less than the sum of the other two.

Therefore, if we let $x$ represent the length of the third side of a triangle with sides $6$ and $19$, we have:

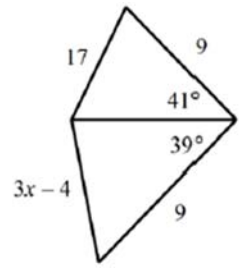
$$19 - 6 < x < 19 + 6$$

$$13 < x < 25$$

There is one solution $(\mathrm{D})$ that fits this requirement; there are three solutions that do **NOT** fit this requirement, which is what the question asks for:

**Answers A, B, C**

2. Find the range of values for $x$.




**Note: never trust the relative sizes of angles and sides in a diagram.**
For example, the two sides with length 9 in this diagram are drawn with different lengths.

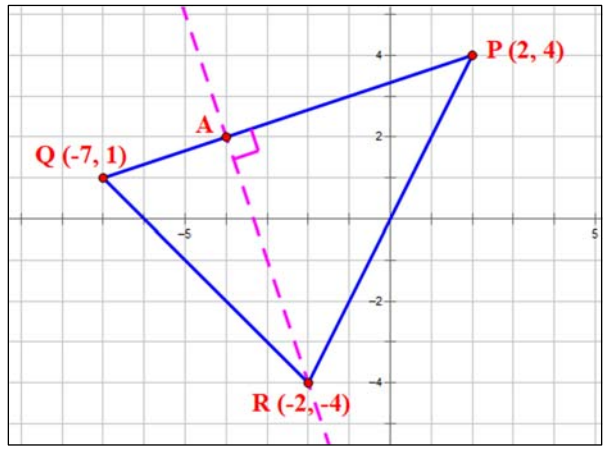
We know two things involving $x$:

- The side labeled $3x - 4$ must be positive. So, $3x - 4 > 0$.
- The two angles shown (39°) and (41°) share two congruent sides (one side with length 9 and one side of unknown length that is shared by the two angles). Therefore, the side opposite the smaller angle must be smaller than the side opposite the larger angle. So, $3x - 4 < 17$.

Combining these two inequalities into a single compound inequality, and solving:

| | |
|---|---|
| Starting inequality: | $0 < 3x - 4 < 17$ |
| Add 4: | $4 < 3x < 21$ |
| Divide by 3: | $\mathbf{\frac{4}{3} < x < 7}$ |

3. $\Delta PQR$ has P(2, 4), Q(-7, 1), and R(-2, -4). Find A if AR is an altitude of $\Delta PQR$.




An altitude of a triangle is a line segment drawn from a vertex to a point on the opposite side (extended, if necessary) that is perpendicular to that side.

Plot the three points given, and draw the altitude from Point R that intersects segment $\overline{PQ}$ at a right angle.

To find the base point of the altitude, we can look at the intersection of the two lines on which Point A lies.

Line containing $\overline{PQ}$:

$$m = \frac{4-1}{2-(-7)} = \frac{3}{9} = \frac{1}{3}$$

Equation: $y - 4 = \frac{1}{3}(x-2)$ using Point P in point-slope form.

Line containing $\overline{AR}$: The slope of $\overleftrightarrow{AR}$ is the negative reciprocal of the slope of $\overleftrightarrow{PQ}$ because the lines are perpendicular.

$$m = -\frac{1}{\frac{1}{3}} = -3$$

Equation: $y + 4 = -3(x+2)$ or $y = -3x - 10$ using Point R in point-slope form.

Find Point A. Substitute $y = -3x - 10$ into $y - 4 = \frac{1}{3}(x-2)$ to get:

| | |
|---|---|
| Equation: | $-3x - 10 - 4 = \frac{1}{3}(x-2)$ |
| Combine constant terms: | $-3x - 14 = \frac{1}{3}(x-2)$ |
| Multiply by 3: | $-9x - 42 = x - 2$ |
| Add $9x + 2$: | $-40 = 10x$ (this can be done in two steps if you prefer) |
| Divide by 10: | $-4 = x$ |

To find $y$, substitute $x = -4$ into the equation of either line:

$$y = -3x - 10$$

$$y = -3(-4) - 10 = 12 - 10 = 2$$

Therefore, **Point A** has coordinates: $(\mathbf{-4, 2})$

4. Write the equation of the perpendicular bisector of CD if C(-4, 3) and D(-8, -9).

Line containing $\overline{CD}$:

$$m = \frac{-9-3}{-8-(47)} = \frac{-12}{-4} = 3$$

Midpoint of $(-4, 3)$ and $(-8, -9)$ is halfway between them: $(-6, -3)$

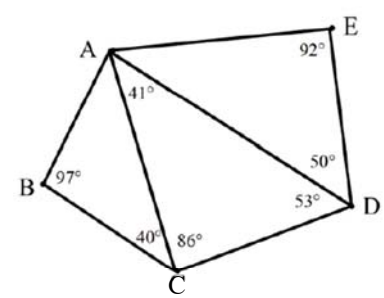
Perpendicular bisector: Slope is the "negative reciprocal" of the slope of $\overleftrightarrow{CD}$ because the lines are perpendicular. Also, $(-6, -3)$ is a point on the perpendicular bisector.

$$m = -\frac{1}{3}$$

**Equation:** $\boldsymbol{y + 3 = -\frac{1}{3}(x+6)}$ **or** $\boldsymbol{y = -\frac{1}{3}(x+6) - 3}$ **or** $\boldsymbol{y = -\frac{1}{3}x - 5}$

point-slope form | $h$-$k$ form | slope-intercept form

5. Identify the longest segment in the diagram shown. Explain your reasoning.




Let's see what we know in each of the triangles. Note that
1) the sum of the angles in each triangle must be 180° and
2) sides across from larger angles in the same triangle are larger.

In $\Delta ABC$:

- $m\angle BAC = 43°$
- $AB < BC < AC$

In $\Delta ACD$:

- $CD < AC < AD$

In $\Delta ADE$:

- $m\angle EAD = 38°$
- $DE < AE < AD$

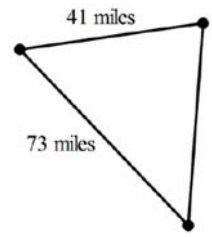
Therefore, the two candidates for longest segment are $\overline{AC}$ and $\overline{AD}$. Looking closer at the above inequalities, we notice that in $\Delta ACD$, we have $AC < AD$. Therefore, the longest segment is: $\boldsymbol{\overline{AD}}$.

6. Given $\Delta WXY$ with W(3, -9), X(2, 11), and Y(-5, 1).
Find the coordinates of D if XD is a median.

Point D is the midpoint of the side of the triangle opposite the given vertex. In this problem, Point X is the vertex in question (it is on the median $\overline{XD}$). So, Point D is the midpoint of the points $W(3, -9)$ and $Y(-5, 1)$.

So, the coordinates of Point D are: $[\,(3, -9) + (-5, 1)] \div 2 = (\mathbf{-1}, \mathbf{-4})$

7. The captain of a boat is planning to travel to three islands in a triangular pattern. What is the possible range for the number of miles round trip the boat will travel?




The silly captain should know the length that is not shown in the diagram, but alas, the captain needs our help. I hope the boat is filled with gas.

The length of the third side of the triangle must be between $73 - 41 = 32$ miles and $73 + 41 = 114$ miles. Therefore:

Round trip miles ($x$): $32 + 41 + 73 < x < 114 + 41 + 73$

$$\mathbf{146 < x < 228}$$

It is interesting to notice that the perimeter must be between double the longest side shown and double the sum of the two sides shown!

8. The radius and height of a tree over a 10-year period can be presented by a directed line segment from point A (2, 5) to point B (10, 15). What ordered pair would represent the radius and height 3 years into this time period? If needed, round to the nearest tenth.

Start at: $(2, 5)$

End at: $(10, 15)$

3 years $= \frac{3}{10} = 0.3$ of the 10 year period.

- This is the factor for $(10, 15)$.
- $(2, 5)$ gets a factor of $1 - 0.3 = 0.7$. The factors must always add to $1$.

Ordered pair @ $t = 3$ years is: $(2, 5) \cdot 0.7 + (10, 15) \cdot 0.3 = (\mathbf{4.4}, \mathbf{8.0})$

Note: an alternative method would be to develop separate equations for the $x$-variable and $y$-variable in terms of time, the $t$-variable. These are called parametric equations, and $t$ is the parameter in the equations. For this problem, the parametric equations would be:

$$\text{variable} = \text{start} + (\text{end} - \text{start}) \cdot \left(\frac{t}{\text{period length in years}}\right)$$

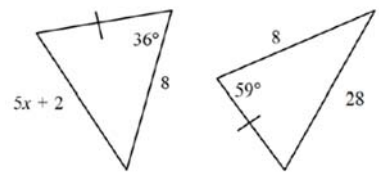
$$x = 2 + (10 - 2) \cdot \left(\frac{t}{10}\right) = 2 + 0.8t$$

$$y = 5 + (15 - 5) \cdot \left(\frac{t}{10}\right) = 5 + t$$

Note that the 10 in the denominator of these equations is the length of time, in years, separating Points A and B.

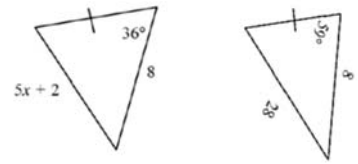
Solve for the required ordered pair by substituting $t = 3$ into these equations.

9. Find the range of values for $x$ in the diagram shown.




First, recognize that $5x + 2 > 0$ or, equivalently, $0 < 5x + 2$.

Next, denote the length of the side marked in both triangles by $y$, just so we can reference it.




If we rotate the triangle on the right to line up with the triangle on the left, it is easier to see that both triangles have sides of length $8$ and $y$ with angles between them.

Since the measure of the angle in the triangle on the left $(36°)$ is less than the one in the triangle on the right $(59°)$, the opposite side on the left must be less than the opposite side on the right.

Therefore, $5x + 2 < 28$.

Combining the two inequalities involving $5x + 2$, we get:

| | |
|---|---|
| Original Inequality: | $0 < 5x + 2 < 28$ |
| Subtract 2: | $-2 < 5x < 26$ |
| Divide by 5: | $-\frac{2}{5} < \boldsymbol{x} < \frac{26}{5}$ |

Note: after you learn Trigonometry, you will be able to reduce this interval substantially.

10. Given triangle ABC such that A(-3, 4), B(7, 1), and C(2, -1). Find D if AD is a median of the triangle.

Point D is the midpoint of the side of the triangle opposite the given vertex.

In this problem, Point A is the vertex in question (it is on the median $\overline{AD}$). So, Point D is the midpoint of the points $B(7, 1)$ and $C(2, -1)$.

So, the coordinates of Point D are: $[\,(7, 1) + (2, -1)] \div 2 = (\mathbf{4.5}, \mathbf{0})$

11. Given a directed segment BC with B(8, -3) and C(-2, 8). Point P is on BC. Find the coordinates of P if the ratio from BP to PC is 3:2.

Start at: $(8, -3)$

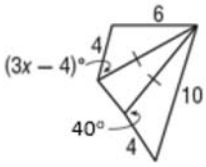
End at: $(-2, 8)$

Factor $= \frac{3}{3+2} = \frac{3}{5} = 0.6$. This factor is based on the ratio of $3:2$ stated in the problem.

- This is the factor for $(-2, 8)$.
- $(8, -3)$ gets a factor of $1 - 0.6 = 0.4$. The factors must always add to $1$.

Coordinates of P are: $(8, -3) \cdot 0.4 + (-2, 8) \cdot 0.6 = (\mathbf{2.0}, \mathbf{3.6})$

12. Write and solve an inequality for $x$.




First, recognize that $3x - 4 > 0$ or $0 < 3x - 4$.

Next, denote the length of the side marked in both triangles by $y$, just so we can reference it.

Both triangles have sides of length $4$ and $y$ with angles between them. Note that this is a necessary condition if we want to compare angles between the two triangles.

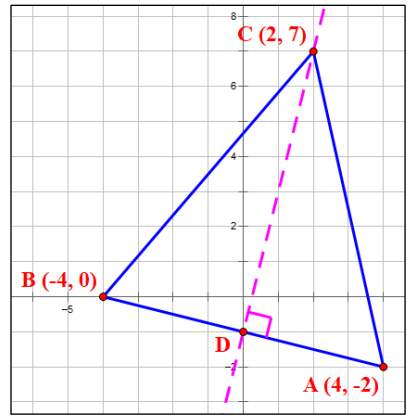
Since the length of the side $(6)$ opposite the angle in question, $(3x - 4)°$, in the upper triangle is less than the length of the side $(10)$ opposite the angle in the lower triangle $(40°)$ we must have: $3x - 4 < 40$.

Combining the two inequalities involving $3x - 4$, we get:

| | |
|---|---|
| Original Inequality: | $0 < 3x - 4 < 40$ |
| Add 4: | $4 < 3x < 44$ |
| Divide by 3: | $\frac{4}{3} < x < \frac{44}{3}$ |

**For #13 – 14,** given triangle ABC such that A(4, -2), B(-4, 0), and C (2, 7).

13. Find D if CD is an altitude of the triangle.




An altitude of a triangle is a line segment drawn from a vertex to a point on the opposite side (extended, if necessary) that is perpendicular to that side.

To find the base point of the altitude, we can look at the intersection of the two lines on which Point D lies.

Line containing $\overline{BA}$:

$$m = \frac{-2-0}{4-(-4)} = -\frac{2}{8} = -\frac{1}{4}$$

Equation: $y = -\frac{1}{4}(x+4)$ using Point B in point-slope form.

Line containing $\overline{CD}$: The slope of $\overleftrightarrow{CD}$ is the negative reciprocal of the slope of $\overleftrightarrow{BA}$ because the lines are perpendicular.

$$m = -\frac{1}{-\frac{1}{4}} = 4$$

Equation: $y - 7 = 4(x-2)$ using Point C in point-slope form, or $y = 4x - 1$

Find Point D. Substitute $y = 4x - 1$ into $y = -\frac{1}{4}(x+4)$ to get:

| | |
|---|---|
| Equation: | $4x - 1 = -\frac{1}{4}(x+4)$ |
| Multiply by 4: | $16x - 4 = -(x+4)$ |
| Distribute the "– " sign: | $16x - 4 = -x - 4$ |
| Add $x + 4$: | $17x = 0$ |
| Divide by 10: | $x = 0$ |

To find $y$, substitute $x = 0$ into the equation of either line:

$y = 4x - 1$

$y = 4(0) - 1 = 0 - 1 = -1$

Therefore, **Point A** has coordinates: $(\mathbf{0}, \mathbf{-1})$

14. Find E if AE is a median of the triangle.

Point E is the midpoint of the side of the triangle opposite the given vertex.

In this problem, Point A is the vertex in question (it is on the median $\overline{AE}$). So, Point E is the midpoint of the points $B(-4, 0)$ and $C(2, 7)$.

So, the coordinates of Point D are: $[\,(-4, 0) + (2, 7)] \div 2 = (\mathbf{-1, 3.5})$

15. Write the equation of the line (in slope-intercept form) that models the distance between (-4, -3) and the line $y = -\frac{2}{7}x + 9$.

This question is asking for the equation of the line perpendicular to $y = -\frac{2}{7}x + 9$ that contains the point $(-4, -3)$.

The perpendicular line will have a slope that is the opposite reciprocal of the original line:

$$m = -\frac{1}{-\frac{2}{7}} = \frac{7}{2}$$

Then, the equation of the perpendicular line (in $h$-$k$ form) is:

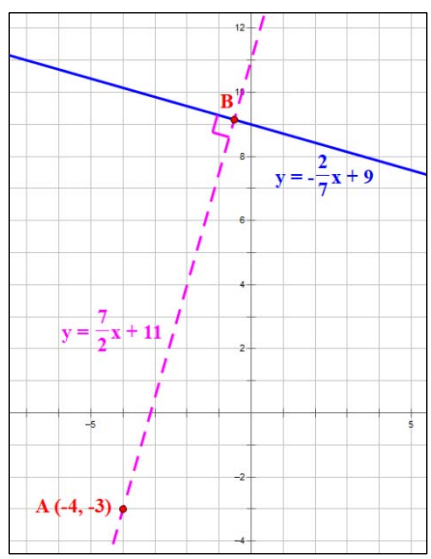
Original equation: $y = \frac{7}{2}(x + 4) - 3$

Distribute the $\frac{7}{2}$: $y = \frac{7}{2}x + 14 - 3$

Collect constant terms: $\boldsymbol{y = \frac{7}{2}x + 11}$

Note: The distance between a point and a line is the length of the segment connecting the point to the line at a right angle. See the diagram to the right.

To calculate the distance between Point A and the line $y = -\frac{2}{7}x + 9$, we would need to find Point B at the intersection of the two lines shown, and then measure the distance between the two points using the distance formula.



16. Find the range of possible values of $x$ if each expression represents the measures of sides of a triangle:
$x+2,\ \ x+6,\ \ 3x-4$

All of the sides must be greater than zero, so we have:

$x > -2,\ x > -6,$ and $x > \frac{4}{3}$ Combining these, we get: $x > \frac{4}{3}$

Notice that we do not know which side is the longest and which side is the smallest, though we know that $x+6$ is always greater than $x+2$. $3x-4$ could be smaller than $x+2$, between $x+2$ and $x+6$, or greater than $x+6$. Let's consider each case separately.

Recall that: 1) the shortest side must be longer than the difference of the other two sides and 2) the longest side must be shorter than the sum of the other two sides, we have:

Case 1: The order of the lengths of the sides is: $(3x-4) < (x+2) < (x+6)$

$(3x-4) > (x+6)-(x+2)$ and $(x+6) < (3x-4)+(x+2)$

$3x-4 > 4$ and $x+6 < 4x-2$

$3x > 8$ and $8 < 3x$

$x > \frac{8}{3}$ and $\frac{8}{3} < x$ Conclude: $x > \frac{8}{3}$

Case 2: The order of the lengths of the sides is: $(x+2) < (3x-4) < (x+6)$

$(x+2) > (x+6)-(3x-4)$ and $(x+6) < (x+2)+(3x-4)$

$x+2 > -2x+10$ and $x+6 < 4x-2$

$3x > 8$ and $8 < 3x$

$x > \frac{8}{3}$ and $\frac{8}{3} < x$ Conclude: $x > \frac{8}{3}$

Case 3: The order of the lengths of the sides is: $(x+2) < (x+6) < (3x-4)$

$(x+2) > (3x-4)-(x+6)$ and $(3x-4) < (x+2)+(x+6)$

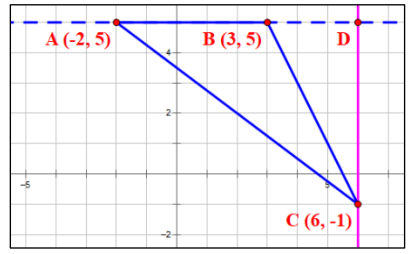
$x+2 > 2x-10$ and $3x-4 < 2x+8$

$12 > x$ and $x < 12$ Conclude: $x < 12$

Other cases: If $(3x-4) = (x+2)$ or $(3x-4) = (x+6)$, one of the above sets of calculations would apply.

Combining all of the cases together, we conclude that $x > \frac{8}{3}$ and $x < 12$. So, $\boldsymbol{\frac{8}{3} < x < 12}$.

17. Triangle ABC has an altitude CD with A(-2, 5), B(3, 5), and C(6, -1). Find the coordinates of D.




An altitude of a triangle is a line segment drawn from a vertex to a point on the opposite side (extended, if necessary) that is perpendicular to that side.

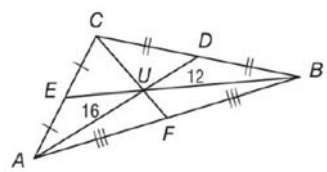
This problem is very straightforward once you graph it. To find the base point of the altitude, we can look at the intersection of the two lines on which Point D lies.

Line containing $\overline{BA}$: $y = 5$

Line containing $\overline{CD}$. $x = 6$ is perpendicular to $y = 5$ and contains $C(6, -1)$.

Therefore, **Point D** has coordinates: $(\mathbf{6}, \mathbf{5})$

18) Given $\Delta ABC$, If $CU = 3x - 2$ and $UF = x + 3$, Find $x$ and CF.




**Centroid**

- The centroid is the intersection of the three medians of a triangle.
- A median is a line segment drawn from a vertex to the midpoint of the side of the triangle that is opposite the vertex.
- The centroid is located 2/3 of the way from a vertex to the opposite side.
- The medians of a triangle create 6 inner triangles of equal area.

In this problem, Point U is the centroid of $\Delta ABC$. Therefore,

$CU = 2(UF)$

$3x - 2 = 2(x + 3)$

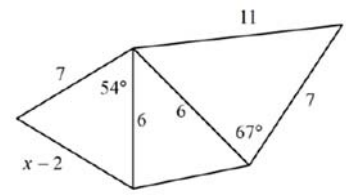
$3x - 2 = 2x + 6$

$\boldsymbol{x = 8}$

Then, $\boldsymbol{CF} = CU + UF = (3x - 2) + (x + 3) = 4x + 1$

$= 4(8) + 1 = \mathbf{33}$

19) Write and solve an inequality for $x$.




Each side must have a positive measure, so: $x - 2 > 0$

$x > 2$

Also, in the triangle on the left, we have:

$7 - 6 < x - 2 < 7 + 6$

$1 < x - 2 < 13$

$3 < x < 15$

Next, both outside triangles have sides of length $6$ and $7$ with angles between them.

Since the measure of the angle in the triangle on the left $(54°)$ is less than the one in the triangle on the right $(67°)$, the opposite side on the left must be less than the opposite side on the right. So, $x - 2 < 11$.

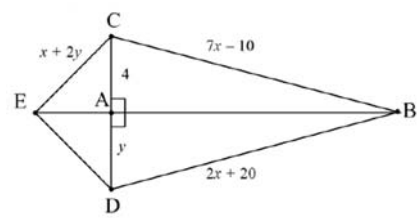
$x < 13$

Putting it all together, we have: $3 < x$, equivalent to $x > 3$, which is more restrictive than $x > 2$, so we use the more restrictive $3 < x$.

We also have: $x < 13$, which is more restrictive than $x < 15$, so we use the more restrictive $x < 13$.

Finally, since $3 < x$ and $x < 13$, we have $\mathbf{3 < x < 13}$

20) Given that BE is the perpendicular bisector of CD, then find the length of ED.




In the diagram, $\overline{CA} \cong \overline{DA}$ by definition of "bisector."

So, $\Delta CAB \cong \Delta DAB$ by SAS, and $\Delta CAE \cong \Delta DAE$ by SAS.

The two hypotenuses of the triangles on the right side of the diagram are congruent. Then,

$7x - 10 = 2x + 20$

$5x = 30$

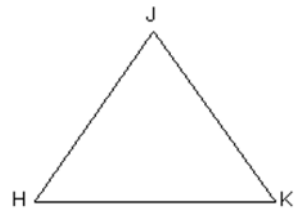
$x = 6$

$CA = DA$, so $y = 4$

Finally, $ED = EC = x + 2y$

$= 6 + 2(4) = \mathbf{14}$

21. **Given:** $\angle H \ncong \angle K$.
**Prove:** $\Delta JHK$ is not isosceles with base $\overline{HK}$.




We will use proof by contradiction on this problem. In proof by contradiction, we assume that the opposite of the conclusion is true, then show that is impossible. This implies that the original assumption is false, so its opposite (what we want to prove) must be true.

| Step | Statement | Reason |
|---|---|---|
| 1 | $\angle H, \angle K$ not congruent | Given |
| 2 | Assume $\Delta JHK$ is isosceles with base $\overline{HK}$. | Assumption intended to lead to a contradiction. |
| 3 | $JK = JH$ | Euclid's definition of isosceles triangle. |
| 4 | $\overline{JK} \cong \overline{JH}$ | Definition of congruent segments. |
| 5 | $\angle H \cong \angle K$ | Angles opposite congruent sides in a triangle are congruent. |
| 6 | Contradiction | We are given $\angle H, \angle K$ are not congruent. |
| 7 | $\Delta JHK$ is not isosceles with base $\overline{HK}$. | Assumption in Step 2 must be false. |